State the Intermediate Value Theorem.

SCORE: ____ / 10 PTS

IF f IS CONTINUOUS ON $[a,b]$
AND N IS BETWEEN $f(a)$ AND $f(b)$
THEN THERE IS A VALUE $c \in (a,b)$
SUCH THAT $f(c)=N$

Let $f(x) = 2x^3 - x^2 - 4$.

SCORE: _____ / 30 PTS

[a] Find $f'(x)$. **No credit will be given for using differentiation shortcuts from chapter 3.**

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^3-(x+h)^2-4-(2x^3-x^2-4)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^3+3x^2h+3xh^2+h^3)-(x^2+2xh+h^2)-4-2x^3+x^2+4}{h}$$

$$= \lim_{h\to 0} \frac{6x^2h+6xh^2+2h^3-2xh-h^2}{h}$$

$$= \lim_{h\to 0} (6x^2+6xh+2h^2-2x-h)$$

$$= 6x^2-2x$$

[b] Find the equation of the tangent to the graph of f at the point where $x = -1$.

$f(-1) = -2-1-4 = -7$ $\quad f'(-1) = 6+2 = 8$

$y+7 = 8(x+1)$

At time t hours, the position of an object moving in a straight line is $p(t) = \dfrac{26+3t}{2+t}$ meters. **SCORE: ____ / 25 PTS**

[a] What is the average velocity of the object from time $t = 3$ to time $t = 8$? **Give the units of your answer.**

$$\frac{p(8)-p(3)}{8-3} = \frac{\frac{50}{10}-\frac{35}{5}}{5} = \frac{5-7}{5} = -\frac{2}{5}\ \text{m/hr}$$

[b] What is the instantaneous velocity of the object at time $t = 3$? **Give the units of your answer.**

$$\lim_{b\to 3} \frac{\frac{26+3b}{2+b}-7}{b-3}$$

$$= \lim_{b\to 3} \frac{26+3b-7(2+b)}{(b-3)(2+b)}$$

$$= \lim_{b\to 3} \frac{12-4b}{(b-3)(2+b)}$$

$$= \frac{-4}{5}\ \text{m/hr}$$

OR

$$\lim_{h\to 0} \frac{\frac{26+3(3+h)}{2+(3+h)}-7}{h}$$

$$= \lim_{h\to 0} \frac{\frac{35+3h}{5+h}-7}{h}$$

$$= \lim_{h\to 0} \frac{35+3h-7(5+h)}{h(5+h)}$$

$$= \lim_{h\to 0} \frac{-4h}{h(5+h)}$$

$$= \frac{-4}{5}\ \text{m/hr}$$

Can the Intermediate Value Theorem be used to prove that the equation $\sec x = -1$ has a solution in the interval $(\frac{\pi}{3}, \frac{4\pi}{3})$? If so, write a proof. If not, explain why not.

SCORE: ____ / 10 PTS

NO. $\sec x = \frac{1}{\cos x}$ IS NOT CONTINUOUS ON $[\frac{\pi}{3}, \frac{4\pi}{3}]$ (AT $\frac{\pi}{2}$)

State the formal definition of "horizontal asymptote". **SCORE: ____ / 5 PTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y=b$ IFF

$$\lim_{x\to\infty} f(x)=b \quad \text{OR} \quad \lim_{x\to-\infty} f(x)=b$$

Find the equation(s) of all horizontal asymptote(s) of $f(x)=\dfrac{2-e^{3x}}{7e^{3x}+5}$. **SCORE: ____ / 15 PTS**

$$\lim_{x\to\infty} \frac{2-e^{3x}}{7e^{3x}+5} = \lim_{x\to\infty} \frac{\frac{2}{e^{3x}}-1}{7+\frac{5}{e^{3x}}} = \frac{0-1}{7+0} = -\frac{1}{7}$$

$\frac{-\infty}{\infty}$

$$\lim_{x\to-\infty} \frac{2-e^{3x}}{7e^{3x}+5} = \frac{2-0}{0+5} = \frac{2}{5}$$

$y=-\frac{1}{7}$ AND $y=\frac{2}{5}$

Let $f(x) = \dfrac{2-x}{x^3 - 3x^2}$.

SCORE: ____ / 20 PTS

[a] Find all discontinuities of f.

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0, 3$

[b] Find the limit of f at each discontinuity.

Each limit should be a number, ∞ or $-\infty$. **Write DNE only if the other possibilities do not apply.**

$\lim_{x \to 0^+} \dfrac{2-x}{x^2(x-3)} = -\infty$ $\quad \dfrac{2}{0^+(-3)}$

$\lim_{x \to 0^-} \dfrac{2-x}{x^2(x-3)} = -\infty$ $\quad \dfrac{2}{0^+(-3)}$

$\lim_{x \to 0} \dfrac{2-x}{x^2(x-3)} = -\infty$

$\lim_{x \to 3^+} \dfrac{2-x}{x^2(x-3)} = -\infty$ $\quad \dfrac{-1}{9(0^+)}$

$\lim_{x \to 3^-} \dfrac{2-x}{x^2(x-3)} = \infty$ $\quad \dfrac{-1}{9(0^-)}$

$\lim_{x \to 3} \dfrac{2-x}{x^2(x-3)}$ DNE

Pat is taking vitamin pills regularly. The time between pills depends on the amount of vitamin in each pill. **SCORE: _____ / 15 PTS**
Let $t = g(v)$, where t is the time between pills (in hours), and v is the amount of vitamin per pill (in grams).

[a] What are the units of $g'(v)$?

HOURS/GRAM

[b] What does $g'(6) = 8$ mean ? **Give the units for all numbers in your answer.**

IF EACH PILL HAS 6 GRAMS OF VITAMIN,
THEN FOR EACH ADDITIONAL GRAM PER PILL,
THE TIME BETWEEN PILLS SHOULD INCREASE 8 HOURS

[c] Is there a value of v_0 for which you would expect $g'(v_0) < 0$? Why or why not ?

NO. IF EACH PILL HAS MORE VITAMINS,
THE TIME BETWEEN PILLS SHOULD INCREASE,
NOT DECREASE

The graph of $f(x)$ is shown on the right.

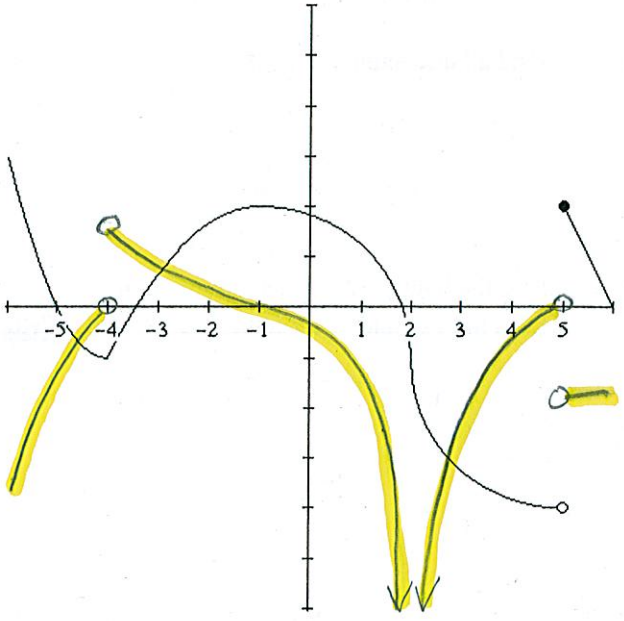
SCORE: ___ / 20 POINTS

[a] List all $x-$values for which $f'(x)$ does not exist.
For each $x-$value, give a **brief** reason why not.

$x = -4$ (CUSP)

$x = 2$ (VERTICAL T.L.)

$x = 5$ (DISCONTINUITY)



[b] Sketch a graph of $f'(x)$ **on the same axes**.